Problems of Science.   By FEDERIGO ENRIQUES.   Authorised Trans-
lation by KATHARINE ROYCE, with an Introductory Note by
JOSIAH ROYCE.   Open Court Company.   Pp. xvi + 392.

THE present work is a translation of the *Problemi della Scienza* of
Prof. Enriques, the eminent Italian mathematician.   It covers very
much the same ground as Poincaré's three books on the philosophy
of science.   It may be divided into five parts; the first is a general
introduction and explanation of the author's position (which he
calls Critical Positivism), the second deals with Logic and its
applicability to the real world, the third deals with geometry, the
fourth with the classical mechanics, and the last with electro-
dynamics and the alterations which it has entailed in the mechanics
of Newton.   The whole work gives an impression of very deep
and wide learning ; Prof. Enriques draws his examples not only
from the subjects in which he is specially an expert, but also from
economics, jurisprudence, and biology.   Unhappily the style is
very heavy, and one can never forget for a moment that one is
reading a translation from a foreign tongue.   The book is also
disfigured by an immense number of notes of exclamation, a stop
which may safely be deleted from all works except novels.   A final
word of general criticism is that although this book is of consider-
able length it deals with so many difficult and important subjects
that the argument is obscure through its condensation even to
persons familiar with the problems under discussion ; to others it
must often be quite unintelligible.   In some few places Prof. Royce
has helped the reader with explanatory notes, and it could be
wished that these were more frequent.   I do not think that
the obscurity of some passages necessarily indicates any confusion
in Prof. Euriques' own mind; it is often merely due to the fact
that he has treated these subjects in special articles elsewhere and
now has to condense his arguments so much that it is difficult to
follow them.

The first part, which introduces us to Critical Positivism, is
largely occupied with a defence of the philosophic doctrine of
relativity.   The argument is that wherever we apparently meet
with an absolute term or an absolute distinction we really only
meet with something that occupies a higher position in a series
than some corresponding term with which we have previously
dealt.   Since the great difficulty of the doctrine of relativity is its
ambiguity it is a pity that Prof. Enriques has not considered the
question quite generally, but has mainly treated special cases of
supposed absolutes and tried to refute their claims.   For instance,
he discusses the claims of certain problems (like the squaring of the
circle) to be absolutely insoluble ; of justice to be an absolute
duty ; of actually infinite numbers, etc.   His conclusion is that
the problems are only insoluble relative to certain means (*e.g.*, the
use of a rule and compass) ; that justice is only absolute in the

sense that it is the ambiguous name given at any moment to the highest duty recognised at that moment; and, so far as I can see, that transfite numbers are either meaningless or mere symbols for the indefinite prolongation of certain finite series of acts. I need scarcely say that many of his particular observations are very valuable; there *is* a pronounced tendency in the human mind to think that any series must have a last term, and this has enabled philosophers to score easy triumphs over the actually infinite by defining it as the last term of an endless series. But on the other hand I cannot see precisely what general conclusion can be deduced from the discussion of a number of claims to absoluteness of such very different kinds, and further I cannot accept all Prof. Enriques' special arguments. For instance justice does not seem to me to be simply the highest duty recognised at any given time; it has a definite content of its own. We may certainly both (*a*) learn more and more clearly what that content is, and (*b*) learn more clearly to what this quality justice applies. And these two processes will generally proceed *pari passu*. But this in no way affects the absoluteness of the duty to be just in the only two senses in which any one maintains it, *viz.*: (1) that no action is right that is not just, and (2) that justice is a perfectly definite quality with an absolutely determinate nature whether or no we have fully analysed that nature and clearly seen precisely what is and what is not just.

Again I cannot see precisely what Prof. Enriques' special argument about the actual infinite is supposed to prove. He says that an actual number cannot be defined as the last term of an infinite series, and further that mere consideration of a series by itself will never prove that it has a limit. (I do not know if he means also to imply that you cannot tell whether an infinite series has a last term by considering it alone.) All this is perfectly true, but I cannot see what bearing it has on the reality of infinite numbers, or how it shows that 'the word "infinite" cannot be applied to any given number or quantity' (p. 15). At best it would show that the concept of a *greatest* infinite number is unsound. And the reference here to the difficulties of Mr. Russell's class *w* seems quite irrelevant. As Prof. Enriques is most unlikely to be under any of the common illusions on these questions I can only say that he seems to me to fail to make clear what exactly he is trying to prove. I am the more convinced of this by the fact that he sometimes speaks as if he believed in the actual infinite, *e.g.* he speaks of a logical analysis being in terms of an infinite number of elements. It is true that he says that these cannot be supposed to be all *given*; but, so far as I can see, 'given' merely means 'thought of in succession,' and the question whether this be psychologically possible seems irrelevant to the actual number of elements.

Prof. Enriques is also concerned to show that there is no absolute

distinction between the subjective and objective. Here he is not referring to the distinction between a mental act and its object (though he sometimes seems to be) but mainly to that between the various processes by which various minds reach a result and the common result. I have two criticisms to make here. (1) His argument seems to be that the subjective can itself be made the object of scientific knowledge :—it helps, *e.g.*, to explain the minimum of cases when scientific predictions are not accurately fulfilled. This is true, but surely two distinct things—mental process and ascertained fact—do not lose their absolute distinction because they are alike in the one respect that both are data for science. And (2) this example shows how difficult it is to collect from Prof. Enriques' special arguments what general principle of relativity he is trying to maintain. If we generalised from this example we should infer that he held that there is no absolute distinction between anything and anything else. And this is either too obvious (in the sense that nothing differs in every respect from anything else) or too absurd (in the sense that there are no definite differences in the world) for any one to maintain.

We now pass to Prof. Enriques' treatment of logic. This is praiseworthy in its insistence on the importance and validity of a system of genuinely formal logic. There are also some excellent remarks on the nature of definition. The definitions of Euclid are not real analyses but serve the same purpose as geometrical models. Fundamental notions can only be defined in this way, or else by postulates. Even nominal definitions are not mere shorthand abbreviations; they mark definite and important groups of entities in a science which are worth treating in detail for their own sake. Just as you may start with elements and axioms and build up complex entities by nominal definition ; so you may be given in experience something which you find you can best treat by assuming it to be a complex built up from certain elements according to certain laws. This is the case in geometry where what is given is lines and surfaces and we find it conducive to our reasoning to regard these as complexes of points. Prof. Enriques thinks it necessary to deal especially with the case where we are led to assume an infinite number of points (as *e.g.* the continuity of lines and surfaces forces us to do). His difficulty, as I understand it, is this. This kind of hypothetical analysis of what is given into entities connected by laws is only helpful if it enables us to suppose that the fundamental entities *might* be given to us in experience and we might build them up by nominal definitions into the complex entities that actually *are* given. Now when your analysis leads to an infinite number of fundamental entities you could not suppose these to be all given in any experience. Prof. Enriques' solution is that as we can know things about any entity of a class without needing to be acquainted with each one separately the infinity of their number need not trouble us. This is undoubtedly

the right type of answer to all psychological difficulties about the infinite.

There are some interesting reflexions on a subject not often touched by logicians, *viz.* the applicability of the Laws of Logic to the existent world. Prof. Enriques concludes that the necessary condition is that there should be great relative invariance in the existent world; logic assumes strict invariance, and, so far as the existent world departs from this, logic becomes less and less applicable to it. I am not perfectly sure that I understand this; but it seems to mean somewhat as follows. Logical operations and deductions are performed on timeless entities, and the existent world is in time. If you take a number of terms and relations in the existent world at a given moment and deduce something further about them by logical reasoning your conclusions will be rigidly applicable to the same things at the given moment; but it will not be rigidly applicable to the things called by the same name and treated for ordinary purposes as the same at some other moment unless they have remained absolutely unchanged during the interval (or, of course, unless they change in accordance with some law which, while it contains time, contains no particular time). There are however certain passages which suggest a much too subjective view of logic. Thus we are told that 'the formal requirements of logical representation express only a psychological fact . . .' and that 'the psychological associations and dissociations which fall within the realm of clear consciousness and volition constitute the fundamental operations of logic'. With the view that these sentences imply I should wholly disagree.

The part of the book devoted to geometry is of great interest and importance, but is often obscured by too great condensation. Prof. Enriques' main effort is to correlate the axioms of projective geometry with sight-space; those of metrical geometry with the space of active touch; and those of Analysis Situs, which underlie both, with general sensibility both of the skin and of the retina. He of course recognises that in ordinary geometry the data of the various senses have all contributed to the 'smoothing act' of the crude spaces of each. This is a very interesting attempt which, as he says, needs a mathematician who is also a psychologist and a physiologist to work it out. He is not able to go enough into detail for me to judge how far he has succeeded.

It is interesting to note that the author holds that the hypotheses of Euclidean and ordinary Non-Euclidean geometry differ more than conventionally, and that the question of their applicability to the existent world can be treated experimentally without a logical fallacy. On the other hand he seems to think that no possible experiment could settle whether the geometry of the real world is Archimedean or non-Archimedean. It would take too long to enter into this question here; much depends on what is meant by the very ambiguous word 'conventional'.

There are two chapters on Mechanics. The first deals with the notions of the classical mechanics. It sees in their comparative success a further verification of Euclid. There is an interesting discussion on time, mass, and the Newtonian laws of motion. Prof. Enriques rejects absolute space and time, and points out that the notion of mass, though it can be reached in Mach's way, can also be reached in several others which do not assume the Third Law of Motion. Force, again, as something about which our muscular sensations tell us, has as good a right to be taken as a datum of mechanics as have the data of any other sense. Prof. Enriques saves Newton's second law from tautology by substituting the law that the *incipient* motion of a particle relative to any frame of reference is in the direction of the force acting on it at that moment, and proportional to its statical measure at that moment relative to the frame in question. He then has to add a law to enable us to pass from incipient to other motions. This is substituted for Newton's first law, and here we have to notice (1) that a special frame of reference has to be chosen (*viz.* one defined by the fixed stars), and (2) that this law has been proved by the electron theory to need modification for velocities large in comparison with that of light.

The whole book is worth reading and may be recommended to those who are pretty familiar with the problems with which it deals.

C. D. BROAD.

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Elementary Logic. By ALFRED SIDGWICK. Cambridge University Press, 1914. Pp. x, 250. Price 3s. 6d. net.

" LOGIC is here treated (1) as a carefully limited subject to get up for an elementary examination ; and (2) as a free study of some of the chief risks of error in reasoning " (p. viii). The book is accordingly divided into two parts entitled " The Old System " and " The Risks of Reasoning ". It would seem, at first sight, difficult or even impossible to harmonise the two aims which Mr. Sidgwick sets before himself. The necessary bond of union, however, is supplied by the conviction that the traditional Logic is a danger to all who think as well as a nuisance to the few who have to pass examinations in it. In this conviction Mr. Sidgwick is at one with Dr. Schiller. As he himself rather warily expresses it : " At the present day we may safely admit that the best reason for knowing something about the old system is in order to see exactly why modern Logic [by 'modern Logic' Mr. Sidgwick means pragmatic logic] has been driven to make certain far-reaching departures from it " (p. viii). In plain words, one of " the chief risks of error in reasoning " lies in the danger of succumbing to ' ideals ' of reason-